MATHEMATICAL MODEL OF DYNAMIC PHENOMENA IN FLUID LAYERS MOVING IN THE SUPPORT OF HYDRODYNAMIC FRICTION OF VIBRATION DAMPER

Galina KHROMOVA, Malika MAKHAMADALIEVA*
Tashkent Institute of Railways Engineering
Adylkhodjaev 1, 100167, Tashkent, Uzbekistan
**Corresponding author.* E-mail: reine_m@mail.ru

Summary. The problem of modeling vibrations of a torsion hydro-friction damper of a locomotive with the support of hydrodynamic friction is studied in the paper based on the application for a Patent of the Republic of Uzbekistan [1]. The objectives of the authors' invention are: to improve the reliability and damping capacity of the damper as a whole, with the provision of horizontal and vertical damping of vibrations and shock loads, which is important at increased speeds of rail transport; to increase the dynamic factor of the system while regulating the damping capacity by creating an additional friction torque to reduce dynamic load on the cantilever section of the shaft fixed to the bogie frame of the vehicle.

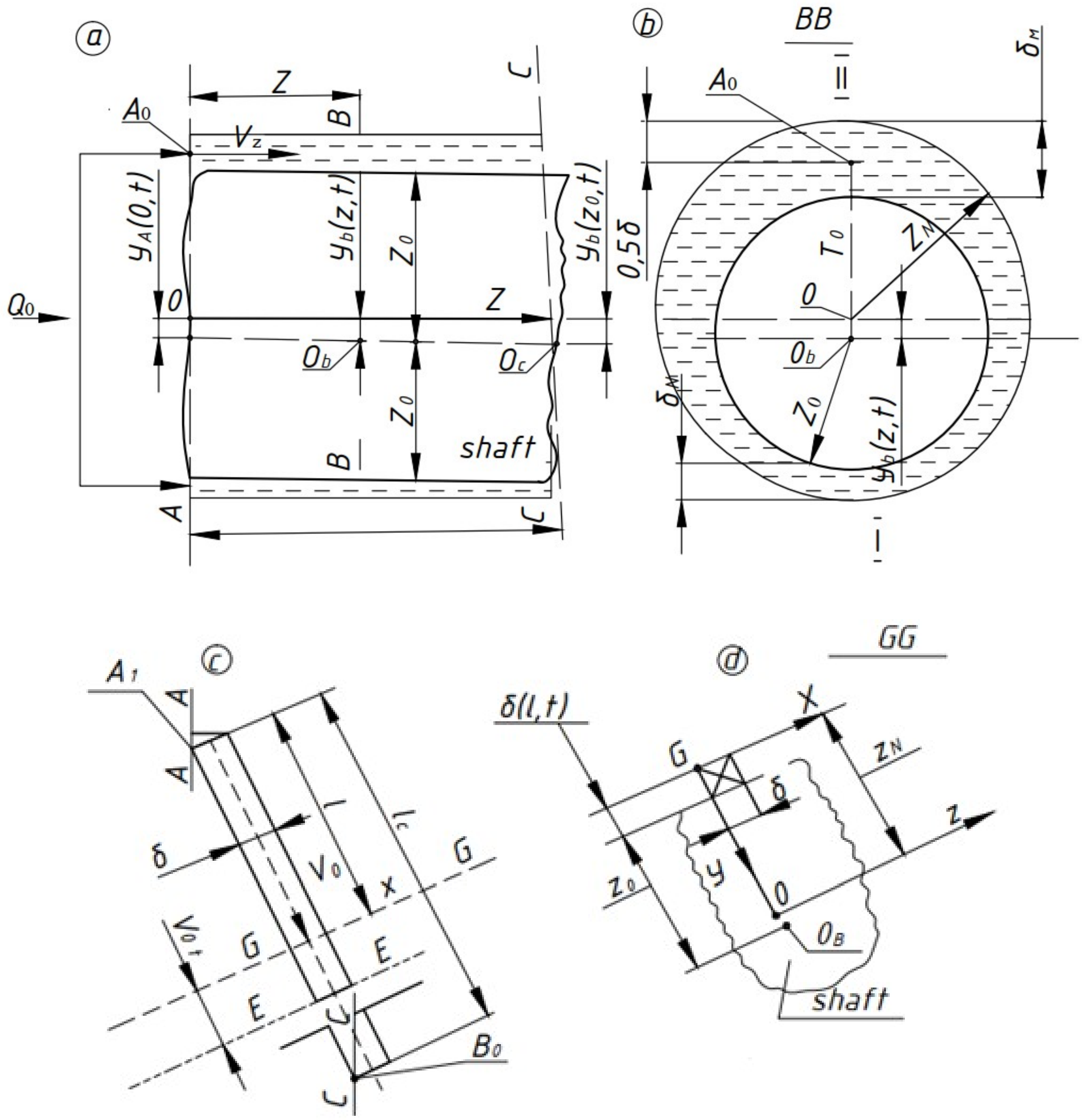
Key words: hydro-friction damper, hydrodynamic friction

INTRODUCTION

Various conflicting requirements are imposed upon hydro-friction dampers of locomotives; for example, they must, on the one hand, ensure reliable operation of the spring suspension system, and on the other hand, at long operation, their dynamic resistance increases as a result of ingress of moisture and various mechanical impurities. In this regard, the creation of a reliable support for hydrodynamic friction and its theoretical substantiation is the most important task in the development of new designs of torsion hydro-friction dampers of locomotives. The solution of this problem will allow authors to simulate dynamic phenomena in the units and parts of the hydro-friction vibration damper of torsion type. This damper presents a shaft rotating in the working fluid. The shaft is considered elastic and has a variable diameter, variable mass and flexural rigidity. The elastic shaft is equipped with movable supports of hydrodynamic friction and has a constant rate of rotation.

METHODS OF CALCULATION

The design scheme used is shown in Figs. 1 a, b in *YOZ* plane and BB section. The lubricant layers inside the support move between cylindrical surfaces of radii $r_н$ and r_0, the elastic strain of the radii is characterized by the function $Y_B(Z, t)$ on $Y_в(Z,t)=Y_1*e^{2\beta Z}+Y_2*e^{4\beta Z}-Y_3(Z)*\cos(3*\omega*t)$. From the surface of the shaft of radius r_0, a reduced concentrated mass T_0 is transferred to the fluid layers; it depends on the forces N_0 and the intensity q_0 (Fig. 1 b, c) of elastic shaft loading. Initial pressure P_A on fluid layers in the end section of an area $S_Z = \pi*d_0*\delta$ and pressure difference ΔP_A after fluid passage through the end section in plane CC (Fig. 1, a) are taken into account. The same lubricant consumption Q_0 through the end sections in AA and CC planes is ensured at a known speed of its motion $V_Z = Q_0 / S_Z$; Q_0 and S_Z being constant for the mode of operation under consideration.



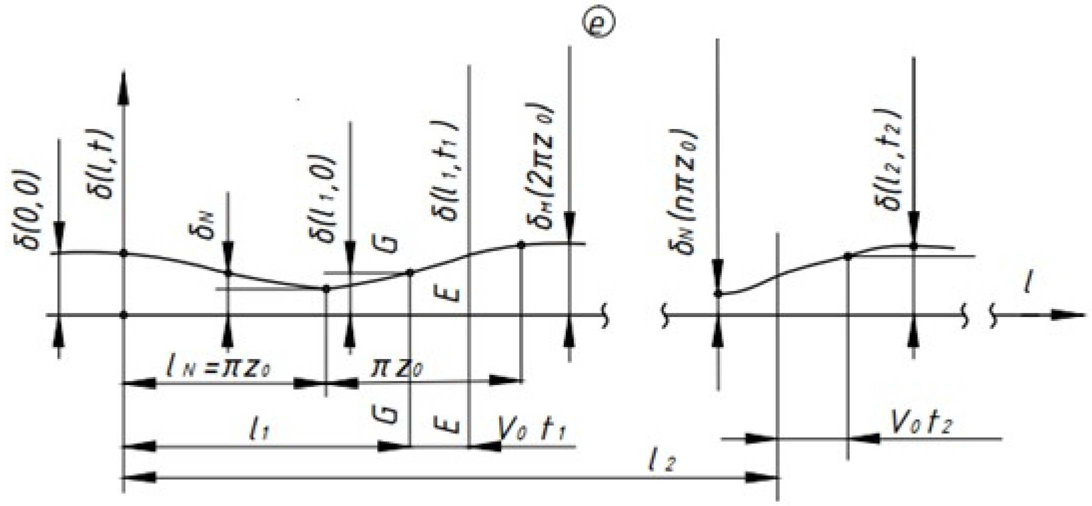


Fig. 1. Design scheme of the support of hydrodynamic friction of a flexible shaft

The surface rotation of radius r_0 of elastic shaft with an angular velocity ω_B = CONST causes the surface layer of lubricant to move with a linear velocity $V_b = \omega_b * r_0$; the lubricant layers on the surface of radius r_n of the support remain stationary. It is assumed that the middle layer between the surfaces of the support and the shaft is characterized by an average peripheral speed $V_0 = 0,5 * V_b$ of lubricant motion. Thus, the motion of the middle layer of lubricant inside the support model (Fig. 1) is characterized by the velocity components V_Z, V_0, by their geometric sum $V_C = \sqrt{V_Z^2 + V_0^2}$ at helix trajectory about the *OZ* axis on the radius $r_0 + 0,5*\delta \approx r_0$ [2].

Introduce the term of the model jet of lubrication of the section δ^2, the centers of gravity of which are located along a helix on a cylindrical surface of radius r_0 ($\delta = r_0 - r_н$).

Inside the model support (Fig. 1) simultaneously operates the CH_S number of identical model jets. Here, $CH_S = \pi * d0 / \delta$. Through the end sections in AA and CC planes of each model jet, a constant lubricant consumption $Q_C = Q_0 / CH_S$ is ensured [3].

At the same time, the conditions are fulfilled for:

- the time of motion of lubricant particles along the *OZ* axis $t_Z = Z_0 / V_Z$;
- the paths of lubricant particles motion along the length l of the model jet, measured from the AA section (Fig. 1, a) are $l_c = T_z \cdot V$ over time T_z;
- the number of complete turns of the helix of the model jet within the length of the shaft support Z_o.

$$i_c \approx \frac{l_c}{\pi d_o} = \frac{T_z \cdot V_o}{\pi d_o} \quad (1)$$

- average pitch between adjacent coils of the jet, measured along the OZ axis

$$b_s = \frac{Z_o}{i_c} = \frac{\pi d_o Z_o}{T_z \cdot V_o} \quad (2)$$

To assess the strain of volume compression of the lubricant layers inside the model jet, we introduce the function *U (l, x, y, t),* which depends on the coordinates:

- $l-¿$ the placement of the section along the length of the helix (counting from A_o point);
- y - the placement of the layer from the cylindrical surface of the radius

$$r_H, y=0 \div \delta_M=\delta \div y_B \quad (3);$$

- x – the placement of a "small jet" in the model jet section counted from the left end surface in the direction of the OZ axis of the end section of the lubricant from the support; Figs. 2.2 c, d show the moving coordinate system YGX in which l, x and y are counted for the model jet, and the movable plane EE, which takes into account the motion of the lubricant at speed V_o.

Specific features of the motion of this lubricant are characterized by pulsed periodic compression when passing through the vertical plane OG to a minimum thickness $\delta_H=\delta-y_B(l,t)$, where the value of l is related to Z coordinate (Fig. 1, a); it takes into account the placement of the flexible shaft sections as well as the parameters of the helix of the model jet (l_c, b_S). The first compression of the lubrication section occurs $l_H=\pi r_o$, followed by $2\pi r_o$. Periodic placement of sections at distances multiple of $2\pi r_o$when passing through the vertical plane OP (Fig. 1, b) is accompanied by an increase in the thickness of the lubricant layer to $\delta_H=\delta+y_B(l,t)$.

Thus, the variable values $y_B(z,t)$ of elastic strains of the flexible shaft cause changes in the values $\delta_H(l,t)$, and $\delta_H(l,t)$ at time $t=0 \div t_z$ or at $l=0 \div l_c$. This phenomenon causes the variability of values of relative compression strain $\frac{\delta(l,t)}{\delta}=\varepsilon(l,t)$ of the lubricant layers and causes pulsed pressure fluctuations in the lubricant volume.

To take into account the elastic compression properties of elementary volumes of lubricant inside a model jet, the modulus of elasticity E_o is used [1]; we separate the elementary volume $V_N=\delta^3$ from the model jet, which has a constant pressure inside the lubricant. In the case of increasing the pressure to P_K the compression of this volume of lubricant is:

$$V_K=(\delta-U_l)(\delta-U_x)(\delta-U_y) \quad (4),$$

where U_l, U_x, U_y are the components of elastic strains of compression in directions of l, x, y axes. Assume that $U_l+U_x+U_y \ll \delta$, which is confirmed by calculations at $E_0>10^8 \frac{Km}{M^2}$ for real lubricants.

In this case, an approximate value can be obtained:

$$V_K \approx \delta^3-\delta^2(U_l+U_x+U_y) \quad (5),$$

And then the relationship

$$\frac{V_K}{V_H}=1-\frac{U_l+U_x+U_y}{\delta} \quad (6).$$

For the adopted scheme of elastic strains of the volumes of the model jet (Fig. 1, c, d), where the GX and GY planes and $y = \delta$ limit elastic strains, the layers and "small jets" may be pressed to these planes by pressure pulses due to relative strains $\varepsilon_y(l,t)=\frac{U_y}{\delta}$ along the GY axis. In this case, decompression of the elementary volume V_K is possible in directions of coordinates $\pm l$. Therefore, the transition to a new volume of strain V_S is presented in the form:

$$V_\varepsilon=(\delta-\varepsilon_y)\cdot\delta(\delta+2\varepsilon_e)\approx\delta^3-\delta^2(\varepsilon_y-2\varepsilon_y) \qquad (7),$$

relative volume strain in the form:

$$\varepsilon_\varepsilon=\frac{\varepsilon_y-2\varepsilon_l}{\delta} \qquad (8).$$

The feed of new lubricant portions in the direction of velocity V_0 or $(-l)$, which will lead to a decrease in $\varepsilon_l$should be taken into account. This also contributes to the presence of pressure P_A in the end section AA (Fig. 1, a). So, the possible range $E_y=\frac{1}{\delta}\cdot[E_y-E_l(2\div1)]$ allows us to accept the average ratio $E_{E_c}=\frac{E_y-1,5E_l}{\delta}$.

A pulsed increase in pressure occurs only at the ratio $E_y>1,5E_l$ in the elementary volume V_N . Taking into account the high propagation velocity of elastic deformation waves, equal to U_E, it is assumed that at pulse relative deformations, E_y exceeds 1,5· E_l and reaches $2E_l$. Therefore, for subsequent calculations of impulse pressures arising in sections with a minimum clearance $\delta_H(l)$ the following formula is used:

$$P_u(l)=E_0\cdot\frac{Y_B(l)}{2\delta}\cdot\left(1-\cos\frac{l}{r_0}\right) \qquad (9),$$

with account of this formula, the function of the intensity of loading of the model jet along the length l can be determined

$$\delta^2\frac{\partial P_u(l)}{\partial l}=\frac{E_0\cdot\delta}{2}\cdot\left[\frac{\partial Y_B(l)}{\partial l}\cdot\left(1-\cos\frac{l}{r_0}\right)+\frac{Y_B(l)}{r_0}\cdot\sin\frac{l}{r_0}\right] \qquad (10),$$

due to pulsed pressure changes in this jet and allowing the replacement of $\frac{l}{r_0}$ by $\omega_0 t$ at $\omega_0=0,5\omega$ (rotation velocity of flexible shaft) and $t=\frac{l}{V_0}$ [2].

Taking into account the limitation of elastic strains along the planes of GX and GY for the motion of the model jet (Fig. 1. c, d, e), we specify the function of the loading intensity of the model jet from Y and X

$$q_u(l,y,x)=\left[q_{uc}(l)\cdot\left(1-\cos\frac{l}{r_0}\right)+q_{us}(l)*\sin\frac{l}{r_0}\right]\cdot\sin\frac{\pi y}{2\delta}\cdot\sin\frac{\pi x}{2\delta} \qquad (11)$$

The substitution of the function $Y_B(z,t)$ on $Y_B(Z,t)=Y_1*e^{2\beta Z}+Y_2*e^{4\beta Z}-Y_3(Z)*\cos(3*\omega*t)$ by approximate one $Y_B(l)$ for the supports is justified by calculations, which showed the relations $Y_1\cdot l^{2\beta Z}\gg Y_2\cdot l^{2\beta_2 Z}\gg Y_3(Z)$. Therefore, for technical calculations, the following presentation is sufficient:

$$Y_B(Z)\approx Y_1\cdot l^{2\beta Z} \qquad (12)$$

The transition to the new function $Y_B(l)$ is performed with account of the ratio of maximum sizes $Z=Z_0, l=l_c$ and equal intensity of the increase of the $l^{2\beta Z_0}=l^{2\beta_c l_c}$, hence

$$\beta_c=\beta\cdot\frac{Z_0}{l_c} \text{ и } Y_B(l)=Y_1\cdot l^{2\beta_c l} \qquad (13)$$

So, $\frac{\partial Y_B(l)}{\partial l} = 2\beta_C \cdot Y_1 \cdot l^{2\beta_c l}$ and

$$q_u(l,y,x) = \frac{1}{2} E_0 \delta Y_1 \cdot l^{2\beta_c l} \cdot \left[2\beta_c \left(1 - \cos\frac{l}{r_0} \right) + \frac{1}{r_0} \cdot \sin\frac{l}{r_0} \right] \cdot \sin\frac{\pi y}{2\delta} \cdot \sin\frac{\pi x}{2\delta} \quad (14)$$

The model jet is considered equivalent to a compressed elastic rod of constant section δ^2 with a semi-infinite length $l_c \gg \delta$ (real ratio $\frac{l_c}{\delta > 10^4}$), which has a compressive rigidity and is loaded with an intensity of external load according to formula (14). This assumption allows the following equation to be used to estimate the volume compression functions of lubricant "small jets" inside a model jet.

$$\frac{S_c}{E_{0g}} \cdot \frac{\partial^2 u}{\partial t^2} - \left(\frac{\partial^2 u}{\partial l^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} \right) = \frac{y_1}{2\delta} \cdot \sin\frac{\pi y}{2\delta} \cdot \sin\frac{\pi x}{2\delta} \cdot l^{2\beta_c l} \cdot \left[2\beta_c \left(1 - \cos\frac{l}{r_0} \right) + \frac{1}{r_0} \cdot \sin\frac{l}{r_0} \right] \quad (15)$$

The first solution to the last equation is found in the form of two independent components

$$U(l,y,x,t) = U_1(t) \cdot l^{2\beta_c l} \cdot \sin\frac{\pi y}{2\delta} \cdot \sin\frac{\pi x}{2\delta} + U_2(l) \cdot \sin\frac{\pi y}{2\delta} \cdot \sin\frac{\pi x}{2\delta} \quad (16)$$

After substituting the partial derivatives of the first component (16) in (15), we obtain the equation

$$\frac{d^2 U_1}{dt^2} + p_u^2 \cdot U_1 = \frac{y_1 E_{0g} \beta_c}{\delta \cdot S_c} \quad (17),$$

where

$$p_u^2 = \frac{E_{og}}{S_c} \cdot \left(\frac{\pi^2}{2\delta^2} - 4\beta_c^2 \right)$$

The initial conditions for (17) correspond to $U_1(0) = U_1$, $\frac{dU_1(0)}{dt} = V_0$, to determine the constant U_1 the condition of equality of lubricant consumption in the initial (p. A), final (p. C) sections and in the model jet (Fig. 1 c) is used:

$$\frac{dU_1(T_z)}{dt} = V_0 \quad (18)$$

The solution of equation (17) by the method of operational calculus is obtained in the form

$$U_1(t) = \frac{2 Y_1 \delta \beta_c}{\pi^2 - 8\delta^2 \beta_c^2} \cdot (1 - \cos p_u t) + U_1 \cdot \cos p_u t + \frac{V_0}{p_u} \cdot \sin p_u t \quad (19)$$

The condition (18) is used:

$$\frac{dU_1(T_z)}{dt} = V_0 = \frac{2 Y_1 \delta \beta_c p_u}{\pi^2 - 8\delta^2 \beta_c^2} \cdot \sin p_u T_z - U_1 p_u \sin p_u T_z + V_0 \cos p_u T_z ,$$

hence

$$U_1 = \frac{2 Y_1 \delta \beta_c}{\pi^2 - 8\delta^2 \beta_c^2} + \frac{V_0 (\cos p_u T_z - 1)}{p_u \cdot \sin p_u T_z} \quad (20)$$

Now substitute the partial derivatives of the second function from (16) into (15) and get the equation

$$\frac{d^2U_2}{dl^2}+\frac{\pi^2}{2\delta^2}\cdot U_2=\frac{Y_1}{2\delta}\cdot l^{2\beta_c l}\cdot\left(\frac{1}{r_0}\cdot\sin\frac{l}{r_0}-2\beta_c\cdot\cos\frac{l}{r_0}\right) \quad (21)$$

which meets the boundary conditions $U_2(0)=U_2,\ \frac{dU_2(0)}{dl}=\frac{P_A}{E_o},$

where P_A is the pressure in the point A (Fig. 1 a) on the lubricant from hydraulic supply to the support, U_2 is a constant determined from the condition of pressure decrease in the point B (Fig. 1, c) to zero

$$\frac{dU_2(l_c)}{dl}=0 \quad (22).$$

The solution of equation (21) is performed by the Carson method of operational calculus.

Let $U_2(l)\longleftarrow U_2(q)$, then

$$\frac{d^2U(l)}{dl^2}\longleftarrow q^2U_2(q)-q^2U_1-q\cdot\frac{P_A}{E_0};$$

$$e^{2\beta_c l}\cdot\sin\frac{l}{r_0}\longleftarrow\frac{q}{r_0\left[(q-2\beta_c)^2+\frac{1}{r_0}\right]}$$

$$e^{2\beta_c l}\cdot\cos\frac{l}{r_0}\longleftarrow\frac{q(q-2\beta_c)}{(q-2\beta_c)^2+\frac{1}{r_0}},$$

The following representation of the solution of equation (21) is obtained

$$U_2(q)=\frac{Y_1(q\lambda_1^2-2\beta_c q^2)}{2\delta(q^2-4\beta_c q+\lambda_1^2)(q^2+\lambda_2^2)}+\frac{q^2U_1+q\frac{P_A}{E_0}}{(q^2+\lambda_2^2)^2} \quad (23)$$

where $\lambda_1^2=\frac{1}{r_0^2}+4\beta_c^2,\ \lambda_2^2=\frac{\pi^2}{2\delta^2},$

To proceed to the originals of function (22), first find the roots $q_{1,2}=2\beta_c\pm\frac{i}{r_0};q_{3,4}=\pm\lambda_2=\pm\frac{i\pi}{\delta\sqrt{2}},$ and then the originals of the representation

$$\frac{q\lambda_1^2-2\beta_c q^2}{(q^2-4\beta_c q+\lambda_1^2)(q^2+\lambda_2^2)}\longrightarrow\frac{r_0\cdot e^{2\beta_c l}}{\left(4\beta_c^2+\lambda_2^2-\frac{1}{r_0^2}\right)^2+\frac{16\beta_c^2}{r_0^2}}\cdot\left\{\sin\frac{l}{r_0}\cdot\left[(\lambda_1^2-4\beta_c^2)\left(4\beta_c^2+\lambda_2^2-\frac{1}{r_0^2}\right)-\frac{8\beta_c^2}{r_0^2}\right]-\frac{2\beta_c}{r_0}\cdot\cos\frac{l}{r_0}\cdot\right.$$

(24)

$$U_2(l)=\frac{Y_1r_0}{2\delta}\cdot¿ \quad (24, a).$$

To determine U_1 the condition (22) is used:

$$U_1=\frac{1}{\lambda_2\cdot\sin\lambda_2 l_c}\cdot\left[\frac{P_A}{E_0}\cdot\cos\lambda_2 l_c+\frac{\partial U_3(l_c)}{\partial e}\right] \quad (25)$$

The solution (24) allows obtaining the components of the pressure function along the directions of the coordinates l,y,x

$$P_l(l,x,y,t)=\frac{\partial(1,51)}{\partial e}\cdot E_0 \quad (26)$$

$$P_y(l,x,y,t)=\frac{\partial(1,51)}{\partial e}\cdot E_0 \tag{27}$$

$$P_x(l,x,y,t)=\frac{\partial(1,51)}{\partial e}\cdot E_0 \tag{28}$$

The second solution to equation (15) is performed by replacing $l=V_0 t$, here $\frac{\partial^2 U}{\partial l^2}=\frac{1}{V_0^2}\cdot\frac{\partial^2 U}{\partial t^2}$ and a new equation with function $U_t(x,y,t)$ is obtained

$$\frac{\partial^2 U_t}{\partial t^2}\left(\frac{1}{V_0^2}-\frac{p_c}{E_{og}}\right)+\frac{\partial^2 U_t}{\partial y^2}+\frac{\partial^2 U_t}{\partial x^2}=\frac{-y_1}{2\delta}\cdot\sin\frac{\pi y}{\delta}\cdot\sin\frac{\pi x}{2\delta}\cdot l^{2\beta_c V_0 t}\cdot\left[2\beta_c(1-\cos\omega_0 t)+\frac{1}{r_0}\cdot\sin\omega_0 t\right], \tag{29}$$

The solution of this equation for the steady-state motion of the model jet is obtained in the form

$$U_t(y,x,t)=U_t(t)\cdot\sin\frac{\pi y}{2\delta}\cdot\sin\frac{\pi x}{2\delta} \tag{30}$$

After substituting the partial derivatives (30) in (29), we obtain

$$\ddot{U}_t(t)-p_t^2 U_t(t)=-e^{2\beta_c V_0 t}\cdot\left[A_1(1-\cos\omega_0 t)+A_2\cdot\sin\omega_0 t\right] \tag{31}$$

where $$p_t^2=\frac{\pi^2\cdot E_o g V_0^2}{2\delta^2(E_{og}-\rho_c V_c^2)}\approx\frac{\pi^2 V_0^2}{2\delta^2},$$

$$A_1=\frac{Y_1 E_o g V_0^2 \beta_c}{\delta(E_{og}-\rho_c V_0^2)},\ A_2=\frac{Y_1 E_o g V_0^2}{2\delta r_0(E_o g-\rho_c V_0^2)},$$

For (30) initial conditions are $U_t(0)=U_{t,}\ \frac{dU_t(0)}{dt}=V_{0,}$ and to determine U_t, a condition of the type (18) is used.

Solution (31) is performed by the method of operational calculus.

Let $U_t(t)\longleftarrow U_t(p)$, then $\ddot{U}_t(t)\leftarrow p^2 U_t(p)-p^2 U_t-pV_0$;

$$e^{2\beta_c V_0 t}\longleftarrow\frac{p}{p-2\beta_c V_0};$$

$$e^{2\beta_c V_0 t}\cdot\cos\omega_0 t\longleftarrow\frac{p(p-2\beta_c V_0)}{(p-2\beta_c V_0)^2+\omega_0^2};$$

$$e^{2\beta_c V_0 t}\cdot\sin\omega_0 t\longleftarrow\frac{p\omega_0}{¿¿¿},$$

the solution representation of (31) is obtained

$$U_t(p)=\left\{A_1\left[\frac{p(p-2\beta_c V_0)}{(p-2\beta_c V_0)^2+\omega_0^2}+\frac{p}{p-2\beta_c V_0}\right]-A_2\frac{p\omega_0}{(p-2\beta_c V_0)^2+\omega_0^2}\right\}\frac{1}{p^2+p_t^2}+\frac{p^2 Ut+pV_0}{p^2+p_t^2} \tag{32}$$

To proceed to the originals of the last function, first determine the roots of the denominator.

$$p_{1,2}=2\beta_C V_0\pm i\omega_0\sqrt{1-\frac{2\beta_c^2 V_0^2}{\omega_0^2}}=2\beta_c V_0\pm i\omega_c;$$

$$\frac{2\beta_c^2 V_0^2}{\omega_0^2}=2\beta_c^2 r_0^2$$

at

$$p_{3,4}=\pm i\,p_t\,,p_{5,6}=\pm 2\beta_c V_0.$$

The transition from representations to the originals of individual functions is conducted

$$\frac{p\left(p-2\beta_c V_0\right)}{\left(p^2+p_t^2\right)\left[\left(p-2\beta_c V_0\right)^2+\omega_0^{\,2}\right]}\longrightarrow\frac{\left(p_t^{\,2}+4\beta_c^{\,2}V_0^{\,2}-\omega_c^{\,2}\right)\cos\omega_c t+4\beta_c V_0\omega_c\sin\omega_c t}{\left[\left(p_t^{\,2}+4V_0^{\,2}\beta_c^{\,2}-\omega_c^{\,2}\right)^2+16\beta_c^{\,2}V_0^{\,2}\omega_c^{\,2}\right]\cdot e^{2\beta_c V_0 t}};$$

$$\frac{p\,\omega_0}{\left(p^2+p_t^2\right)\left[\left(p-2\beta_c V_0\right)^2+\omega_0^{\,2}\right]}\longrightarrow\frac{e^{2\beta_c V_0 t}*\left(p_t^{\,2}+4\beta_c^{\,2}V_0^{\,2}-\omega_c^{\,2}\right)\sin\omega_c t-4\beta_c V_0\omega_c\cos\omega_c}{\left(p_t^{\,2}+4\beta_c^{\,2}V_0^{\,2}-\omega_c^{\,2}\right)^2+16\beta_c^{\,2}V_0^{\,2}\omega_c^{\,2}};$$

$$\frac{p}{\left(p^2+p_t^2\right)\left(p-2\beta_c V_0\right)}\longrightarrow\frac{1}{4\beta_c^{\,2}V_0^{\,2}+p_t^{\,2}}\cdot\left(e^{2\beta_c V_0 t}+\sin p_t t+\frac{2\beta_c V_0}{p_t}\cdot\cos p_t t\right).$$

then the original is obtained (32)

$$U_t(t)=A_1\cdot\left[\frac{\left(з_t^{\,2}+4\beta_c^{\,2}V_0^{\,2}-\omega_c^{\,2}\right)\cos\omega_c t+4\beta_c V_0\omega_c\cdot\sin\omega_c t}{\left(p_t^{\,2}+4\beta_c^{\,2}V_0^{\,2}-\omega_c^{\,2}\right)^2+16\beta_c^{\,2}V_0^{\,2}\omega_c^{\,2}}-1\right]*e^{2\beta_c V_0 t}-A_2*e^{2\beta_c V_0 t}-\frac{\left(p_t^{\,2}+4\beta_c^{\,2}V_0^{\,2}-\omega_c^{\,2}\right)-}{\left(p_t^{\,2}+4\beta_c^{\,2}V_0^{\,2}-\right.}$$

(33)

To determine U_t the following condition is used

$$\frac{dU_t\left(T_z\right)}{dt}=V_0=\frac{d\left(1{,}68\right)}{dt}\quad\text{at } t=T_Z.$$

It is advisable to conduct a comparison of the pressure functions based on (25) for the previously adopted lubricant model in the form of an elastic rod of variable cross section and satisfying the Reynolds equation in our notations:

$$\frac{dp(l)}{dl}=6\mu\omega_0 r_г\cdot\frac{h(l)-h_н}{h^3(l)}=6\mu\omega_0 r_г\cdot\left[\frac{1}{h^2(l)}-\frac{h_н}{h^3(l)}\right]\qquad(34)$$

where the approximate function of the clearance between the shaft and the support, with (5), is taken as $h(l)=\delta_0-\delta_a\cdot\cos\frac{l}{r_г},$ and replace $h_н=\delta_0-\delta_a$ by the average value in the function intervals through $\frac{l}{r_г}=0,\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi.$

$$\left[\frac{h_н}{h(l)}\right]_c=\frac{\delta_0-\delta_a}{S}\cdot\left(\frac{2}{\delta_0+\delta_a}+\frac{2}{\delta_0}+\frac{1}{\delta_0-\delta_a}\right)\qquad(35)$$

So,

$$\frac{dP(l)}{dl}=6\mu\omega_0 r_г\cdot\frac{\alpha}{h^2(l)}=6\mu\omega_0 r_г\cdot\frac{\alpha}{\left(\delta_0+\delta_a\cdot\cos\frac{l}{r_г}\right)^2}\qquad(36)$$

At $\alpha=1-\left[\frac{h_н}{h(l)}\right]c$

The resulting equation allows direct integration:

$$P(l)=6\mu\omega_0 r_г\cdot\alpha\cdot\int_0^l\frac{dl}{\left(\delta_0+\delta_a\cdot\cos\frac{l}{r_г}\right)^2}=\left[\frac{r_г\cdot\delta_a\cdot\sin\frac{l}{r_г}}{\left(\delta_a^{\,2}-\delta_0^{\,2}\right)\left(\delta_0+\delta_a\cdot\cos\frac{l}{r_г}\right)}+\frac{2r_г\delta_0}{\sqrt{\left(\delta_0^{\,2}-\delta_a^{\,2}\right)^3}}\cdot\frac{arctg\left(\delta_0-\delta_a\right)tg\frac{l}{2r_г}}{\sqrt{\delta_0^{\,2}-\delta_a^{\,2}}}\right]\cdot 6\mu\omega$$

(37)

The last formula implies the condition $l_м = \frac{\pi \cdot r_г}{2}$ for the section with the maximum pressure, when $tg\frac{\pi}{2} = \infty$ and $arctg\frac{\delta_0 - \delta_a tg\frac{\pi}{2}}{\sqrt{\delta_0^{\,2} - \delta_a^{\,2}}} = \frac{\pi}{2}$, so

$$P(l_м) = \frac{6\mu\omega_0 r_г^{\,2} \cdot \alpha\pi\delta_0}{\sqrt{\left(\delta_0^{\,2} - \delta_a^{\,2}\right)^3}} \tag{38}$$

1. CONCLUSION

Various hydromechanical systems can be analyzed based on this analytical-numerical method. For example, to analyze the dynamic functioning of hydraulic and hydraulic friction dampers of torsion type for locomotives. Bench dynamic tests have been carried out at the depot "Uzbekistan" — to test a modernized design of a torsion-type hydraulic friction damper.

The authors of the paper have developed the "Instruction on the organization of technological process of overhaul and repair of hydraulic vibration dampers of the KVZ-LIIZHT type", which was sent to the Directorate for the Locomotives Operation of "Uzbekiston Temir Yollari" JSC; the economic effect of its implementation is 58.2 million soum in 2016- 2017 years.

References

1. Khromova G.A., Mukhamedova Z.G., Yutkina I.S., Makhamadalieva M.A. *Hydraulic vibration damper of torsion type.* A positive decision on the patent of the Republic of Uzbekistan for the invention, the application No. IAP 20160113 of 03/29/2016.
2. Khromova G.A., Yutkina I.S., Makhamadalieva M.A. Development of a method for calculating the dynamic strength of EPS hydraulic vibration damper of a torsion type // *In Proc. of the VI International Scientific and Technical Conference "Locomotives. Electric transport. XXI century*", Volume 2, PGUPS, St. Petersburg, November 13–15, 2018 — Pp.38-43.
3. Khromova G.A., Mukhamedova Z.G., Yutkina I.S. *Optimization of dynamic characteristics of emergency restoring rail service cars.* Monograph. ISBN 978-9943-975-96-6. - Tashkent: "Fan va tekhnologiya", 2016. - 253 p.